Simple Example of Condorcet voting counting

Jeffrey Goldberg

August 9, 2002
version 1.3

This is just a simple example of Condorcet vote tabulation to determine a collective ranking. That is, it is a way to go from the individual statements of preferences to a "collective" statement of preferences.

The general idea behind Condorcet voting for each voter to state their preferences among the options, and then for each pair of options consider how many times one option in the pair is prefered to the other. Another way of stating is is that the Condorcet winner is the option that would beat all other options in a head to head race.

Note that this example does *not* address to possible, but rare, cases where there is no "pure" Condorcet winner, that is there there are collective cycles.

I promise you that this is much much easier to do on a blackboard than it is to present on paper. (I also have a computer program for doing it if needed.)

Imagine a vote where there are 6 real options, A–F. Furthermore, you want to distinguish some options as acceptable or not, so we have a dummy option "X". Now imagine that there are 5 voters, p–t.

Each voter is asked to provide a total ordering of their preferences of all the candidates. Suppose the votes are as indicated in table 1.

Now what we then do is simply tally up the number of times each option is preferred to some other option. In table 2 the cells indicate how many times each row was preferred to each column. Note that only the upper (or lower) half of this actually needs to be counted. The other half is predictable.[1]

From Table 2 you calculate the *net gain* of row over column (simply substracting the losses from the wins) to get table 3

[1] This is only true because each voter had to rank all options. If voters were allowed to leave things unranked, you would need to count to get the whole matrix.

p:	$E \succ B \succ C \succ F \succ D \succ X \succ A$
q:	$B \succ F \succ E \succ A \succ D \succ X \succ C$
r:	$B \succ A \succ E \succ F \succ X \succ D \succ C$
t:	$B \succ E \succ C \succ F \succ D \succ X \succ A$
u:	$B \succ E \succ D \succ F \succ C \succ X \succ A$

Table 1: Votes

	A	**B**	**C**	**D**	**E**	**F**	**X**
A		0	1	2	1	1	1
B	5		5	5	4	5	5
C	4	0		2	0	2	3
D	3	0	3		0	1	4
E	4	1	5	5		4	5
F	4	0	3	4	1		5
X	4	0	2	1	0	0	

Table 2: Raw Condorcet count

	A	**B**	**C**	**D**	**E**	**F**	**X**
A		-5	-3	-1	-3	-3	-3
B	5		5	5	3	5	5
C	3	-5		-1	-5	-1	1
D	1	-5	1		-5	-3	3
E	3	-3	5	5		3	5
F	3	-5	1	3	-3		5
X	3	-5	-1	-3	-5	-5	

Table 3: Net Condorcet count

	A	**C**	**D**	**E**	**F**	**X**
A		-3	-1	-3	-3	-3
C	3		-1	-5	-1	1
D	1	1		-5	-3	3
E	3	5	5		3	5
F	3	1	3	-3		5
X	3	-1	-3	-5	-5	

Table 4: Net Condorcet second place count

	A	**C**	**D**	**F**	**X**
A		-3	-1	-3	-3
C	3		-1	-1	1
D	1	1		-3	3
F	3	1	3		5
X	3	-1	-3	-5	

Table 5: Net Condorcet thrid place count

Now what we do is we look for a row with no losses. That is, we look for a row which has no negative signs in it. That row is for B. What this means is that in a one-on-one race between B and each other option, B would win that race. So B is the first place winner. To find the second place, we now remove B from the matrix. (If you were doing this on a blackboard you will simply use an eraser.) But I'll produce the next table (table 4).

Table 4 is identical to 3 except that the B row and column is missing. Now we look for a row in this with no negative values. We find that E is our second place winner. We repeat now to find the thrid place winner by remove the E row and column to get table 5

From table 5 we see that F is our third place winner. And we continue the process to get our next winners (again this is *much* easier to do on a blackboard or on a piece of paper you can dynamically update.

From table 6 we see that D is the 4th place winner. Then C, then X, then A. So this gives a collective ranking of

$$B \succ E \succ F \succ D \succ C \succ X \succ A$$

	A	C	D	X
A		-3	-1	-3
C	3		-1	1
D	1	1		3
X	3	-1	-3	

Table 6: Net Condorcet fourth place count

Variations

If you want you could give special status to the X vote and simply remove from consideration any option which doesn't beat X by at least 3. In that case, after constructing table 2 you would remove C and A from consideration at all and not enter them into further tables. If that were done in this case, the rankings would simply be $B \succ E \succ F \succ D \succ X$. While this variation has some virtues, it can be exploited nastily by minority coalitions of voters to produce a bad result. Without the variation, each voter is best off if they express their true preferences on their ballots. That is the central characteristic of Condorcet voting.

Response to criticism

One thing that people might try to do is argue that if in one persons vote A beats B by a large amount (A is first and B is last) that should somehow count more against B than if B is placed right below A. But the Condorcet method actually does the right thing in weighting. The crucial thing is that *preferences are ordinal and not cardinal.* So if one person weakly says that they prefer A to B while another strongly prefers B to A their votes count the same and cancel each other out (in the head to head race). But the interaction with other rankings does take into account where these things are ranked. So a virtue of this system is that it gives equal voice to preferences and not to rhetoric and reference point. Also the variation describe above (minimum required win over X) can help avoid some conflict.